

LOW Q^2 , LOW x REGION IN ELECTROPRODUCTION – AN OVERVIEW

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Abstract

We summarise existing experimental and theoretical knowledge on the structure function F_2 in the region of low Q^2 and low x . The constraints on the behaviour of structure functions in the limit of $Q^2 = 0$ are listed. Phenomenological low Q^2 parametrisations of the structure functions are collected and their dynamical content is discussed. The high energy photoproduction and nuclear shadowing are also briefly described. Recent update of the low Q^2 , low x experimental data is given.

1 Introduction

Interest in the low Q^2 , low x phenomena in the inelastic lepton–hadron scattering is connected with experimental constraints that the low x region in the present fixed target experiments can only be reached on the expense of lowering the Q^2 values down to 1 GeV² or less (see e.g. ref.[1]). Unified treatment of small- and large Q^2 regions might also be of high practical importance for large Q^2 data analysis. This stems from the fact that in all leptonproduction experiments, including those at HERA, a radiative corrections procedure has to be applied in order to extract the structure functions from the data. The radiative "tails" originating from processes at Q^2 values from the interval $Q_{meas}^2 \geq Q^2 \geq 0$ contribute to measurements at $Q^2 = Q_{meas}^2$ and the knowledge of the structure functions in this Q^2 interval is necessary for the iterative data unfolding procedure [2, 3].

Remembering that due to the conservation of the electromagnetic current the structure function F_2 must vanish in the limit $Q^2 \rightarrow 0$, the Bjorken scaling which holds approximately at high Q^2 cannot be a valid concept at low Q^2 . Theoretical models assuring a smooth transition from the scaling to non-scaling regions and applicable in a wide Q^2 interval are thus necessary for understanding the experimental data and the underlying dynamics. The continuity of the physical processes occuring when passing from low- to high Q^2 is illustrated in fig.1. We shall be predominantly concerned with the region of low Q^2 and high W , i.e. beyond the resonances.

The need for modifying the QCD improved parton model by including contributions to the structure functions behaving as $1/(Q^2)^n$ ($n \geq 1$) is clearly visible in the data [4]. These contributions, called "higher twists" are important at moderate values of Q^2 ($\sim 1 \text{ GeV}^2$). They follow from the operator product expansion [5] and describe effects of the struck parton's interaction with target remnants thus reflecting confinement effects. Clearly the theoretical description unifying the confinement and deep inelastic regions should contain terms of well defined physical origin, corresponding to such contributions.

The purpose of this paper is to collect the existing knowledge about the region of low Q^2 , low x (i.e. Q^2 below 3 GeV^2 and x below 0.03 or so) in a way which could at the same time make it easy to use in practical applications. The paper should be considered as an extension of the comprehensive review of the small x physics [6] towards the more detailed treatment of the low Q^2 problems. We shall be exclusively concerned with **charged** lepton inelastic scattering. The recent review of problems specific for inelastic neutrino (and antineutrino) interactions is presented in the ref. [7]. It is predominantly the structure function F_2 which will be discussed; particular final state structures, like jets, diffractive dissociation, etc. will not be considered.

The content of the paper is as follows. After basic definitions and constraints (section 2) we present theoretical ideas and models which describe the low Q^2 physics (section 3). High energy photoproduction (section 4) is then followed by a description of phenomenological parametrisations of structure functions (section 5). Special attention is given to dynamical models of a low Q^2 behaviour of F_2 (section 6). Nuclear shadowing is described in section 7 and finally an update of experimental data is given in section 8. Section 9 contains conclusions and outlook.

2 Basic definitions and constraints

Kinematics of inelastic charged lepton scattering is defined in fig.2. One photon exchange approximation is assumed throughout this paper. The imaginary part of the forward Compton scattering amplitude of the virtual photon is defined by the tensor $W^{\mu\nu}$ (see e.g. [8]):

$$W^{\mu\nu}(p, q) = \frac{F_1(x, Q^2)}{M} \left(-g_{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{F_2(x, Q^2)}{M(p \cdot q)} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) \quad (1)$$

In this equation $Q^2 = -q^2$ where q^2 is the square of the four-momentum transfer, $x = Q^2/(2p \cdot q)$ the Bjorken scaling variable and M is taken as the proton mass. The invariant quantity $p \cdot q$ is related to the energy transfer ν in the target rest frame, $p \cdot q = M\nu$. The invariant mass of the electroproduced hadronic system, W , is then $W^2 = M^2 + 2M\nu - Q^2$. Often one denotes $W^2 \equiv s$.

The deep inelastic regime is defined as a region where both Q^2 and $2M\nu$ are large and their ratio, x , is kept fixed. At Q^2 smaller than few GeV^2 , x can probably no longer be interpreted as a momentum of a struck parton but it remains a convenient variable for displaying the data. The functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$ are the structure functions of the target. For a nuclear target it will be assumed that the structure functions are

normalised to the number of nucleons in the target nucleus and denoted F_i^A , $i = 1, 2$ (except for the deuteron where a symbol F_2^d will be used). The tensor $W^{\mu\nu}$ satisfies the current conservation constraints:

$$\begin{aligned} q_\mu W^{\mu\nu} &= 0 \\ q_\nu W^{\mu\nu} &= 0 \end{aligned} \quad (2)$$

This follows from the fact that $W^{\mu\nu}$ is related to the matrix element of the product of the electromagnetic current operators $j_{em}^\mu(x)$:

$$W^{\mu\nu} \equiv Im T^{\mu\nu} \propto Im \int d^4z \exp(iqz) \langle p | T j_{em}^{\mu+}(z) j_{em}^\nu(0) | p \rangle \quad (3)$$

where the symbol 'T' denotes time ordering.

Let us rearrange eq.(1) in order to display explicitly the potential kinematical singularities of the tensor $W^{\mu\nu}$ at $Q^2 = 0$:

$$\begin{aligned} W^{\mu\nu}(p, q) &= -\frac{F_1}{M} g^{\mu\nu} + \frac{F_2}{M(p \cdot q)} p^\mu p^\nu + \\ &\quad \left(\frac{F_1}{M} + \frac{F_2 p \cdot q}{M q^2} \right) \frac{q^\mu q^\nu}{q^2} - \frac{F_2 p^\mu q^\nu + p^\nu q^\mu}{M q^2} \end{aligned} \quad (4)$$

These singularities cannot be real and appear only as artifacts of the way we wrote up $W^{\mu\nu}$. In order to eliminate them we have to impose the following conditions on the structure functions F_i in the limit $Q^2 \rightarrow 0$:

$$F_2 = O(Q^2) \quad (5)$$

$$\frac{F_1}{M} + \frac{F_2 p \cdot q}{M q^2} = O(Q^2) \quad (6)$$

These conditions have to be fulfilled for arbitrary ν . They will play important role for the parametrisations of the structure functions at low Q^2 .

The differential electroproduction cross section is expressed in the following way by the structure functions F_i :

$$\frac{d^2\sigma(x, Q^2)}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{Mxy}{2E} \right) \frac{F_2(x, Q^2)}{x} + \left(1 - \frac{2m^2}{Q^2} \right) y^2 F_1(x, Q^2) \right] \quad (7)$$

where E denotes the energy of the incident lepton in the target rest frame, m is the electron (muon) mass, $y = \nu/E$ and α is the electromagnetic coupling constant.

Instead of F_1 the structure function $R(x, Q^2)$ defined as

$$R(x, Q^2) = \frac{\sigma_L}{\sigma_T} = \frac{(1 + 4M^2 x^2 / Q^2) F_2}{2x F_1} - 1 \quad (8)$$

is often used where σ_L and σ_T denote the cross sections for the longitudinally and transversally polarised virtual photons respectively. The differential cross section (7) then reads

$$\frac{d^2\sigma(x, Q^2)}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4} \frac{F_2}{x} \left[1 - y - \frac{Mxy}{2E} + \left(1 - \frac{2m^2}{Q^2} \right) \frac{y^2 (1 + 4M^2 x^2 / Q^2)}{2(1 + R)} \right] \quad (9)$$

Real photons are only transversally polarised and therefore σ_L and R vanish when $Q^2 \rightarrow 0$. This vanishing follows from eqs (5) and (6). The function R is related to the frequently used longitudinal structure function $F_L(x, Q^2)$ via $R = F_L/2xF_1$, and

$$F_L(x, Q^2) = \left(1 + \frac{4M^2x^2}{Q^2}\right) F_2 - 2xF_1. \quad (10)$$

At large Q^2 , F_L is directly sensitive to the gluon distribution function, which plays a crucial role in the interactions at small x , [5].

3 Basic theoretical concepts relevant for the small Q^2 region

In the leading $\log Q^2$ approximation of perturbative QCD which is applicable in the high Q^2 region the structure function $F_2(x, Q^2)$ is directly related to the quark- and antiquark momentum distributions, $q_i(x, Q^2)$ and $\bar{q}_i(x, Q^2)$:

$$F_2(x, Q^2) = x \sum_i e_i^2 [q_i(x, Q^2) + \bar{q}_i(x, Q^2)] \quad (11)$$

where 'i' denotes quark flavours and e_i are the quark charges. At high Q^2 the quark and antiquark distributions exhibit the approximate Bjorken scaling mildly violated by the QCD logarithmic corrections. The evolution of these distributions with Q^2 is described by the Altarelli-Parisi equations [9]. These equations as well as the relation (11) acquire corrections proportional to $\alpha_s(Q^2)$ in the next-to-leading $\log Q^2$ approximation.

Systematic analysis of the structure functions in the Bjorken limit can be done using the operator product expansion of the electromagnetic currents (cf. eq. 3), [5]. This expansion leads to the expansion of the structure functions in the inverse powers of Q^2 :

$$F_2(x, Q^2) = \sum_{n=0}^{\infty} \frac{C_n(x, Q^2)}{(Q^2)^n} \quad (12)$$

where the functions $C_n(x, Q^2)$ depend weakly (i.e. logarithmically) on Q^2 . Various terms in this expansion are referred to as leading ($n = 0$) and higher ($n \geq 1$) twists. The "twist" number is defined in such a way that the leading one is equal to two and higher ones correspond to consecutive even integers.

Thus the right hand side of the equation (11) with approximate Bjorken scaling of quark distributions corresponds to the "leading twist" contribution to the F_2 . For moderately large values of Q^2 (Q^2 of the order of a few GeV^2) contributions of the "higher twists" may become significant. Contrary to the common opinion the higher twists are only corrections to the leading (approximately scaling) term (11) in the large Q^2 region. Thus they cannot correctly describe the low Q^2 (i.e. nonperturbative) region since the expansion (12) gives a divergent series there. In particular it should be noted that the individual terms in this expansion violate the constraint (5). In order to correctly describe this region the (formal) expansion has to be summed beforehand, at large Q^2 , and then continued to the region of $Q^2 \sim 0$. This is automatically provided by certain models like the Vector Meson Dominance (VMD) model. To be precise the VMD model together

with its generalisation which gives (approximate) scaling at large Q^2 can be represented in a form (12) for sufficiently large Q^2 .

The VMD model is a quantitative realisation of the experimental fact that the photon interactions are often similar to these of a hadron [10, 11]. The structure function F_2 is in this model represented by:

$$F_2 \left[x = Q^2/(s + Q^2 - M^2), Q^2 \right] = \frac{Q^2}{4\pi} \sum_v \frac{M_v^4 \sigma_v(s)}{\gamma_v^2 (Q^2 + M_v^2)^2} \quad (13)$$

The quantities $\sigma_v(s)$ are the vector meson–nucleon total cross sections, M_v is the mass of the vector meson v and γ_v^2 can be related in the standard way to the leptonic width of the v [10]:

$$\frac{\gamma_v^2}{\pi} = \frac{\alpha^2 M_v}{3\Gamma_{e^+e^-}}. \quad (14)$$

If only the finite number of vector mesons is included in the sum (13) then the F_2 vanishes as $1/Q^2$ at large Q^2 . Therefore it does not contain the "leading twist" term. The scaling can be introduced by including the infinite number of vector mesons in the sum. This version of the VMD is called the Generalised Vector Meson Dominance (GVMD) model [10, 11]. The heavy mesons contribution is directly related to the structure function in the scaling region.

In practical applications to the analysis of experimental data which extend to the moderate values of Q^2 one often includes the higher twists corrections in the following simplified way:

$$F_2(x, Q^2) = F_2^{LT}(x, Q^2) \left[1 + \frac{H(x)}{Q^2} \right] \quad (15)$$

where the F_2^{LT} is the leading twist contribution to F_2 and $H(x)$ is determined from fit to the data. This simple minded expression may not be justified theoretically since in principle the higher twist terms, i.e. functions $C_n(x, Q^2)$ for $n \geq 1$ in eq.(12) evolve differently with Q^2 than the leading twist term.

At high energies the Regge theory [12] is often used to parametrise the cross sections as the functions of energy W . The high energy behaviour of the total cross sections is then given by the following expression:

$$\sigma_t(W) = \sum_i \beta_i (W^2)^{\alpha_i - 1} \quad (16)$$

where α_i are the intercepts of the Regge poles and β_i denote their couplings. The intercepts α_i are the universal quantities, i.e. they are independent of the external particles or currents and depend only on the quantum numbers of the Regge poles which are exchanged in the crossed channel. The Regge pole exchange is to a large extent a generalisation of the particle exchange and formally describes a pole of the partial wave amplitude in the crossed channel in the complex angular momentum plane. The Regge pole corresponding to the vacuum quantum numbers is called pomeron. It is expected to have the highest intercept, close to unity. It is a phenomenologically established fact that the energy dependence of the total hadronic and photoproduction cross sections can be described by two contributions: the (effective) pomeron with intercept $\alpha_P=1.08$ and the Reggeon with the intercept α_R close to 0.5 [13]. Since bulk of the cross sections comes from the "soft" processes one usually refers to the phenomenological pomeron having its intercept slightly above unity as the "soft" pomeron. On the other hand in the leading logarithmic

approximation of perturbative QCD one finds the pomeron with intercept significantly above unity [14, 15]. Since perturbative QCD is only applicable for the description of the "hard" processes one refers to this QCD pomeron as the "hard" pomeron. Its relation to and interplay with the phenomenologically determined "soft" pomeron is still not fully understood. It should also be emphasised that the pomeron with intercept above unity leads to violation of unitarity at asymptotic energies. The unitarity is restored by multiple scattering absorptive (or shadowing) terms.

The Regge parametrisation of the total cross sections implies the following parametrisation of the electroproduction structure function $F_2(x, Q^2)$:

$$F_2(x, Q^2) = \sum_i \beta_i(Q^2) (W^2)^{\alpha_i - 1} \quad (17)$$

It is expected to be valid in the high energy limit and for $W^2 \gg Q^2$. In this limit $x \simeq Q^2/W^2$ and $x \ll 1$. Therefore the Regge parametrisation (17) implies the following small x behaviour of $F_2(x, Q^2)$:

$$F_2(x, Q^2) = \sum_i \tilde{\beta}_i(Q^2) x^{1 - \alpha_i} \quad (18)$$

where

$$\tilde{\beta}_i(Q^2) = (Q^2)^{\alpha_i - 1} \beta_i(Q^2) \quad (19)$$

At low Q^2 the functions $\beta_i(Q^2)$ should obey the constraint (5), i.e. $\beta_i(Q^2) = O(Q^2)$ for $Q^2 \rightarrow 0$.

4 High energy photoproduction

The low Q^2 region should join smoothly the photoproduction limit, $Q^2 = 0$. In this limit the following relation between the total photoproduction cross section $\sigma_{\gamma p}(E_\gamma)$ and the structure function F_2 holds:

$$\sigma_{\gamma p}(E_\gamma) = \lim_{Q^2 \rightarrow 0} 4\pi^2 \alpha \frac{F_2}{Q^2} \quad (20)$$

This limit should be taken at fixed $\nu = E_\gamma$ where E_γ is taken as photon energy in the laboratory frame. Several structure function parametrisations use the photoproduction total cross section as an additional constraint.

When considering photoproduction it is conventional to decompose the total cross section $\sigma_{\gamma p}$ into two parts:

$$\sigma_{\gamma p} = \sigma_{VMD} + \sigma_{part} \quad (21)$$

and then

$$\sigma_{part} = \sigma_{direct} + \sigma_{anomalous} \quad (22)$$

In this equation σ_{VMD} denotes the cross section corresponding to the VMD photon-hadron interaction mechanism while σ_{part} to the partonic mechanism respectively. The partonic component of the cross section is next decomposed into two terms: the "direct" term which reflects the photon interactions with the partonic constituents of the hadron and the "anomalous" term which corresponds to the interactions of the partons – constituents of the photon with partonic constituents of the hadron. In the latter case the photon

coupling to its constituents is point-like. The main feature of the partonic mechanism is that it corresponds to the (semi) hard interactions which can be described by perturbative QCD. The VMD part on the other hand contains both hard as well as the soft components. The former comes from the hard interaction of the partons which are the constituents of the vector meson. Unlike the "anomalous" part this term cannot be described by perturbative QCD alone since its description requires knowledge of the (nonperturbative) parton distributions in the vector meson. The anomalous term together with the hard part of the VMD contribution represent the point-like interaction of the partonic constituents of the photon. In the literature the corresponding events are therefore called the "resolved" photon events [16, 17, 18, 19].

It follows from (20) that the total photoproduction cross section can be obtained from the extrapolation of F_2/Q^2 to $Q^2 = 0$. The partonic component σ_{part} (i.e. the sum of the "direct" and the "anomalous" terms) can be identified with the difference between the result of the extrapolation and the VMD part [18]. The most important distinction between various mechanisms can be done through the analysis of the final states which will not be discussed here (see [18]).

The photoproduction total cross section, like the hadron-hadron total cross sections exhibits an increase with the increasing photon energy, [20, 21, 22], see fig.3. This increase can be well described by the "soft" pomeron contribution with its intercept equal to 1.08 [13] (see also sec. 3). It is also well described by the extrapolation of the parametrisation by Abramowicz et al., [23]. Both predictions are shown in fig. 3. Another possible description of the total cross section increase is provided by the (mini)jet production, i.e. production of jets with relatively low p_T (beginning from 1–2 GeV or so). The minijet production is described by the hard scattering of partons coming from the photon and from the proton respectively. The energy dependence of the cross section reflects to a large extent the small x behaviour of the parton (i.e. mostly gluon) distributions in a proton and in a photon. This follows from the kinematics of the hard parton-parton scattering:

$$x_1 x_2 W_{\gamma p}^2 \geq 4p_T^2 \quad (23)$$

where $x_{1,2}$ denote the momentum fractions carried by partons and $W_{\gamma p}$ is the total CM energy. For increasing $W_{\gamma p}$ (and for fixed p_T^2), x_1 and x_2 may assume smaller values. The magnitude of the cross section is sensitive to the magnitude of the minimal value of p_T . It can be shown that the multiple scattering (or absorptive) corrections are extremely important here and they slow down significantly the increase of the total cross section with energy [18, 19, 24, 25]. Possible prediction for the total cross section energy dependence which follows from the minijet production picture are shown in fig. 3 (see [22] for the details).

The global quantity like the total cross section is not capable to discriminate between different models and the detailed structure of final states may be crucial here. The relevant Monte Carlo event generator for the minijet model with multiple scattering is discussed in [26].

5 Phenomenological parametrisations of structure functions

There exist several phenomenological parametrisations of the structure functions which incorporate the $Q^2 \rightarrow 0$ constraints (cf. sec.2) as well as the Bjorken scaling behaviour at large Q^2 [27, 28, 29, 30, 31, 23, 32]. Certain parametrisations [23, 29] also contain the (QCD motivated) scaling violations. The parametric form of the corresponding Q^2 dependence, however, is not constrained at large Q^2 by the Altarelli–Parisi evolution equations, i.e. those parametrisations are not linked with the conventional QCD evolution. Nor is the low Q^2 behaviour related to the explicit vector meson dominance, known to dominate at low Q^2 , [10].

A parametrisation used by CHIO Collaboration [27] to fit their data is a combination of a simple parton model (valence quarks) and the GVMD spirit approach (sea quarks):

$$F_2(x, Q^2) = P_3(2 + g_3)x(1 - x)^{1+g_3} + P_5 \frac{14}{9} \frac{4 + g_5}{5 + g_5} (1 - x)^{1+g_5} \frac{Q^2}{Q^2 + m_0^2} \quad (24)$$

where $g_3 = g_{03} + \varepsilon$, $g_5 = g_{05} + \varepsilon$, $\varepsilon = \kappa \ln[(Q^2 + m_0^2)/m_0^2]$ and $P_3, P_5, g_{03}, g_{05}, \kappa$ and m_0 are free parameters. This parametrisation is obsolete and has been mentioned here for historical reasons only.

A parametrisation by Brasse et al. [28] concerns the resonance region $0.1 \leq Q^2 \leq 6 \text{ GeV}^2$, $1.11 \leq W \leq 1.99 \text{ GeV}$, cf. fig.1. The virtual Compton scattering cross section is assumed as:

$$\sigma(Q^2, W, \epsilon) = \sigma_T(Q^2, W) + \epsilon \sigma_L(Q^2, W) \quad (25)$$

where the parameter ϵ in the formula (25) denotes the degree of polarisation of the virtual photon and the cross sections $\sigma_T(Q^2, W)$ and $\sigma_L(Q^2, W)$ are related in a standard way to the structure functions F_2 and F_L :

$$F_2 = \frac{Q^2}{4\pi^2\alpha} (\sigma_T + \sigma_L) \quad (26)$$

$$F_L = \frac{Q^2}{4\pi^2\alpha} \sigma_L \quad (27)$$

The cross section $\sigma(Q^2, W, \epsilon)$ is parametrised in the following way:

$$\ln(\sigma/G^2) = a(W) + b(W) \ln \frac{|\mathbf{q}|}{|\mathbf{q}_0|} + c(W) \left| \ln \frac{|\mathbf{q}|}{|\mathbf{q}_0|} \right|^{d(W)} \quad (28)$$

where \mathbf{q} is the three momentum transfer to the hadronic system, i.e. $|\mathbf{q}| = \sqrt{Q^2 + \nu^2}$, $|\mathbf{q}_0|$ is the value of $|\mathbf{q}|$ for $Q^2=0$, i.e. $|\mathbf{q}_0| = (W^2 - M^2)/2M$ and $G^2(Q^2)$ is the dipole form factor of the nucleon, i.e.:

$$G^2(Q^2) = \left(\frac{1}{1 + Q^2/0.71 \text{ GeV}^2} \right)^2 \quad (29)$$

The parameters of $a(W), b(W), c(W)$ and $d(W)$ were obtained from the fit to the data in different bins of W . Their values are tabulated in ref.[28].

A complete parametrisation of F_2^d including the resonance region was obtained by the NMC [29]. In the resonance region F_2^d was fitted to the data from SLAC

[33] taking only the $\Delta(1232)$ resonance into account. Outside the resonance region a QCD based parametrisation was used to describe the data of CHIO [27], SLAC [34], BCDMS [35] and EMC NA28 [36]. For this purpose, the structure function was parametrised as:

$$F_2^d(x, Q^2) = [1 - G^2(Q^2)][F^{dis}(x, Q^2) + F^{res}(x, Q^2) + F^{bg}(x, Q^2)] \quad (30)$$

where F^{dis} and F^{res} are the contributions from the deep inelastic and resonance regions respectively and F^{bg} describes the background under the resonance. The nucleon electromagnetic form factor is given by eq. (29); the factor $1-G^2$ in eq. (30) suppresses F_2 at low values of Q^2 where elastic scattering on the nucleon dominates.

The contribution from the deep inelastic region was parametrised as

$$F^{dis}(x, Q^2) = \left[\frac{5}{18} \frac{3}{B(\eta_1, \eta_2 + 1)} x_w^{\eta_1} (1 - x_w)^{\eta_2} + \frac{1}{3} \eta_3 (1 - x_w)^{\eta_4} \right] S(x, Q^2) \quad (31)$$

where $x_w = (Q^2 + m_a^2)/(2m\nu + m_b^2)$ with $m_a^2 = 0.351 \text{ GeV}^2$ and $m_b^2 = 1.512 \text{ GeV}^2$. The quantity B is the Euler's beta function and η_1, \dots, η_4 are linear functions of the variable \bar{s} ,

$$\eta_i = \alpha_i + \beta_i \bar{s}, \quad (32)$$

where

$$\bar{s} = \ln \frac{\ln[(Q^2 + m_a^2)/\Lambda^2]}{\ln[(Q_0^2 + m_a^2)/\Lambda^2]} \quad (33)$$

with $Q_0^2 = 2.0 \text{ GeV}^2$ and $\Lambda = 0.2 \text{ GeV}$. The constants $\alpha_1, \dots, \alpha_4$ and β_1, \dots, β_4 were free parameters in the fit.

The factor $S(x, Q^2)$ in eq.(31) suppresses F^{dis} in the resonance region close to the single pion production threshold:

$$S(x, Q^2) = 1 - e^{-a(W - W_{thr})}, \quad (34)$$

with $W_{thr} = 1.03 \text{ GeV}$ and $a = 4.177 \text{ GeV}^{-1}$.

The form adopted for the contribution from the resonance region was

$$F^{res}(x, Q^2) = \alpha_5^2 G^{3/2} e^{-(W - m_\Delta)^2/\Gamma^2}, \quad (35)$$

with $m_\Delta = 1.232 \text{ GeV}$, $\Gamma = 0.0728 \text{ GeV}$ and α_5 , a free parameter in the fit, was equal to 0.89456. This parametrisation takes into account only the $\Delta(1232)$ contribution; higher mass resonances are neglected.

The background under the resonance region was parametrised as

$$F^{bg}(x, Q^2) = \alpha_6^2 G^{1/2} \xi e^{-b(W - W_{thr})^2}, \quad (36)$$

where

$$\xi = \sqrt{\frac{[(W + c)^2 + M^2 - m_\pi^2]^2}{4(W + c)^2} - M^2} \quad (37)$$

with $b = 0.5 \text{ GeV}^{-1}$ and $c = 0.05 \text{ GeV}$. The parameter α_6 , left free in the fit, was equal to 0.16452.

$$F_2(x, Q^2) = A(x) \cdot \left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right]^{B(x)} \cdot \left[1 + \frac{C(x)}{Q^2} \right];$$

$$Q_0^2 = 20 \text{ GeV}^2, \quad \Lambda = 250 \text{ MeV};$$

$$A(x) = x^{a_1}(1-x)^{a_2} [a_3 + a_4(1-x) + a_5(1-x)^2 + a_6(1-x)^3 + a_7(1-x)^4];$$

$$B(x) = b_1 + b_2x + b_3/(x + b_4);$$

$$C(x) = c_1x + c_2x^2 + c_3x^3 + c_4x^4.$$

Parameter	proton	deuteron
a_1	-0.1011	-0.0996
a_2	2.562	2.489
a_3	0.4121	0.4684
a_4	-0.518	-1.924
a_5	5.967	8.159
a_6	-10.197	-10.893
a_7	4.685	4.535
b_1	0.364	0.252
b_2	-2.764	-2.713
b_3	0.0150	0.0254
b_4	0.0186	0.0299
c_1	-1.179	-1.221
c_2	8.24	7.50
c_3	-36.36	-30.49
c_4	47.76	40.23

Table 1: *The parametrisation of F_2^p and F_2^d [1]. This function is strictly valid only in the kinematic range of the NMC, SLAC and BCDMS data.*

The parametrization (30) of the function F_2^d is valid from $Q^2 \sim 0$ up to about 200 GeV^2 and from $x = 0.003$ up to 0.7. However, the results of the fit given in [29] are obsolete since the new F_2 measurements by the NMC [1] which were not included in the fit differ by up to 30% from the former world data. When used later in the radiative corrections procedure the deep inelastic part of F_2 was refitted while the parametrisation of the resonances and of the background was kept fixed. Two types of the former were used: 8- and 15-parameter functions. The 15-parameter function and the fitted values of its coefficients are given in Table 1 [1].

A low Q^2 parametrisation of F_2^p was obtained by Donnachie and Landshoff [30]. In this parametrisation the parton model is extrapolated down to the region of low Q^2 (including the photoproduction) respecting the constraints discussed in sec.2. The parametrisations of the sea- and valence quark distributions at small values of x are based on Regge theory (cf. sec.3) and correspond to the pomeron intercept equal approximately 1.08 and reggeon intercept about 0.5. The structure function F_2^p is given

by the conventional parton model formula:

$$F_2^p = x \left[\frac{4}{9}(u + \bar{u}) + \frac{1}{9}(d + \bar{d}) + \frac{1}{9}\lambda(s + \bar{s}) \right] \quad (38)$$

where u, \bar{u}, \dots are the parton densities and λ measures the reduced strength of the pomeron's coupling to strange quarks ($\lambda \approx 0.6$). The individual parton densities are related in the following way to the sea- and valence quark distributions, $S(x)$ and $V(x)$ respectively.

$$\begin{aligned} xu(x) &= S(x) + 2V(x), & xd(x) &= S(x) + V(x), & xs(x) &= S(x) \\ x\bar{u}(x) &= x\bar{d}(x) = x\bar{s}(x) = S(x) \end{aligned} \quad (39)$$

The densities $S(x)$ and $V(x)$ are parametrised as follows:

$$\begin{aligned} S(x) &= 0.17x^{-0.08}(1-x)^5, \\ V(x) &= 1.33x^{0.56}(1-x)^3 \end{aligned} \quad (40)$$

The functions $S(x)$ and $V(x)$ are next multiplied by the factors $\phi_s(Q^2)$ and $\phi_v(Q^2)$, respectively, where

$$\begin{aligned} \phi_s(Q^2) &= \left(\frac{Q^2}{Q^2 + 0.36} \right)^{1.08} \\ \phi_v(Q^2) &= \left(\frac{Q^2}{Q^2 + 0.85} \right)^{0.44} \end{aligned} \quad (41)$$

The powers are chosen so as to make F_2^p vanish like Q^2 as $Q^2 \rightarrow 0$. This parametrisation has been recently extended and modified [31] to include in more detail the heavy quark contribution and the region of large x . Instead of the scaling variable x one uses:

$$\xi_i = x \left(1 + \frac{\mu_i^2}{Q^2} \right) \quad (42)$$

where the index 'i' denotes charm, strange and light quarks. Strange and charm distributions are as follows:

$$\begin{aligned} xs(x, Q^2) &= C_s \frac{Q^2}{Q^2 + a_s} \xi_s^{-\epsilon} (1 - \xi_s)^7 \\ xc(x, Q^2) &= C_c \frac{Q^2}{Q^2 + a_c} \xi_c^{-\epsilon} (1 - \xi_c)^7 \end{aligned} \quad (43)$$

where $C_s \approx 0.22$, $C_c = 0.032$, $a_s = 1 \text{ GeV}^2$, $a_c = 6.25 \text{ GeV}^2$ and $\epsilon = 0.0808$. Parameters $\mu_{s,c}^2$ defining the variables $\xi_{s,c}$ (cf. eq. (42)) are: $\mu_s^2 = 1.7 \text{ GeV}^2$ and $\mu_c^2 = 16 \text{ GeV}^2$.

Parametrisation of the light quark distributions is more involved, i.e. different functional forms obtained from a fit to the data are used for ξ smaller and greater than a certain ξ_0 which is a free parameter in the fit. The valence quark distributions are parametrised as below:

$$\begin{aligned} xu_v(x, Q^2) &= U(\xi)\psi(Q^2) \\ U(\xi) &= B_u \xi^{0.4525} \quad \xi < \xi_0 \end{aligned} \quad (44)$$

and

$$\begin{aligned}
xd_v(x, Q^2) &= D(\xi)\psi(Q^2) \\
D(\xi) &= B_d \xi^{0.4525} & \xi < \xi_0 \\
D(\xi) &= \beta_d \xi^{\lambda_d} (1 - \xi)^4 & \xi > \xi_0
\end{aligned} \tag{45}$$

The parameter μ^2 defining the variable ξ for light quarks is $\mu^2 = 0.28 \text{ GeV}^2$. Parameters $B_{u,d}$, $\beta_{u,d}$ and $\lambda_{u,d}$ are fixed by requiring that the two functional forms defining $U(\xi)$ and $D(\xi)$ join smoothly at ξ_0 and by imposing the number sum rules. The function ψ is $\psi(Q^2) = Q^2/(Q^2 + b)$. The light sea quark distributions are:

$$\begin{aligned}
q_S(x, Q^2) &= C \xi^{-0.0808} \Phi(Q^2) & \xi < \xi_0 \\
q_S(x, Q^2) &= \gamma \xi^{\lambda_s} (1 - \xi)^7 \Phi(Q^2) & \xi > \xi_0
\end{aligned} \tag{46}$$

where $\Phi(Q^2) = Q^2/(Q^2 + a)$. The parameters C, a and b are fixed by fitting to the data on photo- and electroproduction. The model contains also a Regge-like term in the sea quark distributions which behaves like $\xi^{0.4525}$ at small x . It also contains an extra "higher twist" term in the F_2 which is parametrised as below:

$$ht(x, Q^2) = D \frac{x^2(1 - \xi)^2}{1 + Q^2/Q_0^2} \tag{47}$$

The values of other parameters defining the model are given in [31]. The parametrisation is valid for $0 < Q^2 < 10 \text{ GeV}^2$ and for $x \geq 0.008$.

The function $F_2^p(x, Q^2)$ resulting from the parametrisation of Donnachie and Landshoff is shown in fig.4 together with the data.

A parametrisation proposed by Abramowicz et al., [23], is a result of a fit to the following structure function data sets: SLAC [34], BCDMS [35] and EMC NA28 [36] and the then available photoproduction cross section. The fitted function is based on the parton model with the QCD motivated scaling violation appropriately extrapolated to the region of low Q^2 (including the photoproduction). The Regge ideas are used in the parametrisation.

The structure function F_2 is decomposed into two terms, F_2^P and F_2^R , corresponding to pomeron and reggeon exchange,

$$F_2 = F_2^P + F_2^R \tag{48}$$

with each of them expressed as follows:

$$F_2^r = \frac{Q^2}{Q^2 + m_0^2} C_r(t) x_r^{a_r(t)} (1 - x)^{b_r(t)} \tag{49}$$

where

$$\frac{1}{x_r} = \frac{2M\nu + m_r^2}{Q^2 + m_r^2} \tag{50}$$

and $r = P, R$ stands either for pomeron or for reggeon. The argument of the functions $C_r(t)$, $a_r(t)$ and $b_r(t)$ is defined as:

$$t = \ln \left(\frac{\ln[(Q^2 + Q_0^2)/\Lambda^2]}{\ln(Q_0^2/\Lambda^2)} \right) \tag{51}$$

The functions C_R, b_R, a_R and b_P which increase with Q^2 were assumed to be of the form:

$$p(t) = p_1 + (p_1 - p_2)t^{p_3} \quad (52)$$

while the functions C_P and a_P were assumed in the form:

$$p(t) = p_1 + (p_1 - p_2) \left(\frac{1}{1 + t^{p_3}} - 1 \right) \quad (53)$$

The parameters m_0^2, m_r^2, Q_0^2 as well as the parameters a_i, b_i and C_i (separately for the pomeron and the reggeon terms, $i=1,2,3$) were obtained from the fit to the data.

The fit of Abramowicz et al. is valid from $Q^2=0$ up to the highest Q^2 values obtained in the fixed target experiments, excluding the resonance region, $W < 1.75$ GeV. The fit given in [23] should, however, be updated since the recent NMC data on F_2 [1] were not included. The structure function $F_2^p(x, Q^2)$ which follows from this representation is shown in fig.5.

A parametrisation by Capella et al. [32] is based on a fit to the fixed target data as well as to the most recent large Q^2 data from HERA, [37, 38] which show the significant increase of F_2 with decreasing x . It also uses the data on the photoproduction cross section including the highest energy results from HERA [21, 22]. This parametrisation assumes a Regge form of F_2 at small x and counting rules at large x . The leading small x behaviour corresponding to the pomeron is parametrised by the Q^2 dependent effective pomeron intercept $1+\Delta(Q^2)$ which interpolates between the "soft" pomeron at $Q^2=0$ and the "hard" one at large Q^2 . The unusual Q^2 dependence of the effective pomeron intercept is meant to describe in a phenomenological way the absorptive corrections to F_2 which are expected to be strong at low Q^2 values. The small x behaviour corresponding to the Reggeon contribution is parametrised in a conventional way, cf. eqs (18,19). To be precise the structure function $F_2(x, Q^2)$ is parametrised in the following form:

$$\begin{aligned} F_2(x, Q^2) = & Ax^{-\Delta(Q^2)}(1-x)^{n(Q^2)+4} \left(\frac{Q^2}{Q^2+a} \right)^{1+\Delta(Q^2)} \\ & + Bx^{1-\alpha_R}(1-x)^{n(Q^2)} \left(\frac{Q^2}{Q^2+b} \right)^{\alpha_R} \end{aligned} \quad (54)$$

The first term in (54) corresponds to the pomeron contribution where the effective pomeron intercept is parametrised as:

$$\Delta(Q^2) = \Delta_0 \left(1 + \frac{2Q^2}{Q^2+d} \right) \quad (55)$$

with $\Delta_0 \sim 0.08$ that corresponds to the intercept of the "soft" pomeron. The second term in (54) corresponds to the reggeon contribution and at large Q^2 can be identified with the valence quarks. The pomeron part corresponds to the sea quark contribution at large Q^2 . The function $n(Q^2)$ is parametrised as follows:

$$n(Q^2) = \frac{3}{2} \left(1 + \frac{Q^2}{Q^2+c} \right) \quad (56)$$

Parameters A, B, a, b, c, d are obtained from the fit to the data. The parametrisation (54) is used for moderately small and small Q^2 (including photoproduction, i.e. $Q^2=0$). The region of large Q^2 is described by the combination of this parametrisation with the Altarelli-Parisi evolution equations.

6 Dynamical models of the low Q^2 behaviour of F_2

In the previous section we have presented various parametrisations of the structure functions. Those parametrisations although being motivated dynamically were not linked with conventional QCD evolution at large Q^2 (except of that by Capella et al., [32]). Nor was the low Q^2 dynamics explicitly taken into account. In this section we present parametrisations which will contain the QCD evolution. One of them will also include the VMD dynamics, dominating at low Q^2 .

Dynamically calculated structure functions by Glück, Reya and Vogt [39, 40, 41] extend the QCD evolution equations down to the very low Q^2 region ($Q^2 < 1 \text{ GeV}^2$). The parton distributions are calculated by evolving from the input valence-like distributions provided at the very low scale $\mu^2 = 0.3 \text{ GeV}^2$ and both the leading and next-to-leading order approximations are used. The valence-like shape at the scale μ^2 is assumed not only for the valence quarks but also for gluon and sea quark distributions:

$$xg(x, \mu^2) = Ax^\alpha(1-x)^\beta, \quad x\bar{q}(x, \mu^2) = A'x^{\alpha'}(1-x)^{\beta'} \quad (57)$$

The parametrisation of the valence quarks is provided at the large scale $Q^2 = 10 \text{ GeV}^2$ and its form at $Q^2 = \mu^2$ is obtained from the backward QCD evolution. Parameters describing the parton distributions come from a fit to the data at large Q^2 and are also constrained by the sum rules. As a result of the QCD evolution the gluon and sea quark distributions (multiplied by x) become immediately singular at small x for $Q^2 > \mu^2$. In the leading twist approximation the absolute value of the slope in x of these distributions and so the absolute value of the slope in x of the F_2 grows with increasing Q^2 . Naturally the model which is based on perturbative QCD cannot be extended into the region of very low $Q^2 \leq \Lambda^2$ where Λ is the QCD scale parameter since the (perturbatively calculated) QCD coupling $\alpha_s(Q^2) \rightarrow \infty$ for $Q^2 \rightarrow \Lambda^2$. Moreover the measurable quantities like the F_2 may contain non-negligible higher twist contribution in the region of moderately small Q^2 , cf. eq. (12). Presence of higher twists in an observable should not affect the QCD evolution of the leading twist parton distribution since partonic distributions of different twists evolve separately. Surprisingly the QCD evolution turns out to be stable and gives positive definite parton distributions down to the very small scale $Q^2 \cong (2\Lambda)^2$. Also it turns out that in the next-to-leading order approximation of perturbative QCD, the F_2 is more stable than the distributions of partons [41]. Although the higher twist contribution can in principle be important in the low Q^2 region, the leading twist F_2 calculated within the model has recently been successfully confronted [40] with the low x and moderate Q^2 (Q^2 equal few GeV^2) NMC data. The high Q^2 , low x results of HERA are reasonably described too.

The explicit forms of the parton distributions' parametrisations are as follows:

$$xv(x, Q^2) = Nx^a(1 + A\sqrt{x} + Bx)(1-x)^D \quad (58)$$

for valence quarks

$$xw(x, Q^2) = \left[x^a(A + Bx + Cx^2) \left(\ln \frac{1}{x} \right)^b + s^\alpha \exp \left(-E + \sqrt{E's^\beta \ln \frac{1}{x}} \right) \right] (1-x)^D \quad (59)$$

for gluon and light sea quarks and

$$xw'(x, Q^2) = \frac{(s - s_{w'})^\alpha}{\left(\ln \frac{1}{x} \right)^a} (1 + A\sqrt{x} + Bx)(1-x)^D \exp \left(-E + \sqrt{E's^\beta \ln \frac{1}{x}} \right) \quad (60)$$

for heavy sea quarks, where s in the formulae (59–60) is:

$$s = \ln \frac{\ln[Q^2/\Lambda^2]}{\ln[\mu^2/\Lambda^2]}. \quad (61)$$

In these equations A, B, C, D, E, E', a and b are low order polynomials of s . These polynomials as well as values of remaining parameters can be found in [39].

Electroproduction structure function F_2 in the low Q^2 , low x region by Badełek and Kwieciński [42, 43]. Contributions from both the parton model with QCD corrections suitably extended to the low Q^2 region and from the low mass vector mesons were taken into account. The former contributions results from the large Q^2 structure function analysis [44, 45] which includes the recent F_2 measurements by the NMC [1].

The starting point is the Generalised Vector Meson Dominance (GVMD) representation of the structure function $F_2(x, Q^2)$ [42]:

$$\begin{aligned} F_2[x = Q^2/(s + Q^2 - M^2), Q^2] &= \frac{Q^2}{4\pi} \sum_v \frac{M_v^4 \sigma_v(s)}{\gamma_v^2 (Q^2 + M_v^2)^2} + Q^2 \int_{Q_0^2}^{\infty} dQ'^2 \frac{\Phi(Q'^2, s)}{(Q'^2 + Q^2)^2} \\ &\equiv F_2^{(v)}(x, Q^2) + F_2^{(p)}(x, Q^2) \end{aligned} \quad (62)$$

The function $\Phi(Q^2, s)$ is expressed as follows:

$$\Phi(Q^2, s) = -\frac{1}{\pi} \text{Im} \int^{-Q^2} \frac{dQ'^2}{Q'^2} F_2^{AS}(x', Q'^2) \quad (63)$$

The asymptotic structure function $F_2^{AS}(x, Q^2)$ is assumed to be given. It may be obtained from the QCD structure function analysis in the large Q^2 region. By construction, $F_2(x, Q^2) \rightarrow F_2^{AS}(x, Q^2)$ for large Q^2 . The second term in (62) can be looked upon as the extrapolation of the (QCD improved) parton model for arbitrary Q^2 . The first term corresponds to the low mass vector meson dominance part since the sum extends over the low mass vector mesons. Contribution of vector mesons heavier then Q_0 is included in the integral in (62). Choosing the parameter $Q_0^2 > (M_v^2)_{max}$ where $(M_v)_{max}$ is the mass of the heaviest vector meson included in the sum one explicitly avoids double counting when adding two separate contributions to the structure function. Note that Q_0 should be smaller than the mass of the lightest vector meson not included in the sum. The representation (62) is written for fixed s and is expected to be valid at $s \gg Q^2$, i.e. at low x but for arbitrary Q^2 .

In [43] the representation (62) for the partonic part $F_2^{(p)}(x, Q^2)$ was simplified as follows:

$$F_2^{(p)}(x, Q^2) = \frac{Q^2}{(Q^2 + Q_0^2)} F_2^{AS}(\bar{x}, Q^2 + Q_0^2) \quad (64)$$

where

$$\bar{x} = \frac{Q^2 + Q_0^2}{s + Q^2 - M^2 + Q_0^2} \equiv \frac{Q^2 + Q_0^2}{2M\nu + Q_0^2} \quad (65)$$

Simplified parametrisation (64) connecting $F_2^{(p)}(x, Q^2)$ to F_2^{AS} by an appropriate change of the arguments of the latter possesses all the main properties of the second term in (62). First of all it is evident that $F_2^{(p)}(x, Q^2) \rightarrow F_2^{AS}(x, Q^2)$ for large Q^2 . Moreover the parametrisation of $F_2^{(p)}$ defined by (64) preserves the analytic properties of the second term in the eq.(62).

It should be stressed that apart from the parameter Q_0^2 which is constrained by physical requirements described above the representation (62) does not contain any other free parameters except of course those which are implicitly present in the parametrisation of parton distributions defining F_2^{AS} .

The proton structure function F_2^p calculated from the representation (62) with $F_2^{(p)}(x, Q^2)$ given by the eq.(64) is shown in fig.6. In an "approach to scaling" (i.e. in the Q^2 dependence of F_2 at Q^2 less than 1 GeV² or so) visible in fig.6 the change of curvature comes from the factor Q^2 in eq. (62) and the magnitude of this change, particularly at $Q^2 \ll 1$ GeV², is controlled by the vector meson contribution, a non-trivial test of this mechanism. Although the vector meson contribution $F_2^{(v)}$ dominates in the very low Q^2 region ($Q^2 < 1$ GeV²) the partonic component $F_2^{(p)}$ still gives a significant contribution (i.e. at least 20% – 30%) there. At $Q^2 = 10$ GeV² the structure function F_2 calculated from the model differs from $F_2^{AS}(x, Q^2)$ by less than 3% .

The x dependence at fixed Q^2 reflects both the energy dependence of σ_v and the x -dependence of F_2^{AS} . Expectations coming from Regge theory are incorporated in the parametrisations of the $\sigma_v(s)$, [42] and of the input parton distributions at the reference scale $Q^2 = 4$ GeV², [44]. $F_2(x, Q^2)$ increases with decreasing x and that reflects the increase of the total cross sections $\sigma_v(s)$ with increasing s as well as the increase of $F_2^{(p)}(x, Q^2)$ with decreasing x . The increase of $F_2^{(p)}(x, Q^2)$ with decreasing x is weaker at small Q^2 than the similar increase in the large Q^2 region. The s dependence of $\sigma_v(s)$ is also relatively weak in the relevant region of s , [42]. As the result the x dependence of $F_2(x, Q^2)$ is relatively weak for low Q^2 .

The model is applicable in the small x region ($x < 0.1$) and for arbitrary value of Q^2 , however the way of parametrising the total vector meson–nucleon cross sections imposes additional constraint: $\nu > 10$ GeV.

A parametrisation by Schuler and Sjöstrand [18] extends those by Abramowicz et al. and by Donnachie and Landshoff including explicitly the damping of structure functions at small x which is implied by screening effects in QCD [15, 46] and taking into account the logarithmic scaling violations at large Q^2 . Extrapolations towards the region of small Q^2 are done applying the same damping factors as in refs. [23, 30, 31]. In the region of low x the Regge parametrisation is used. In practice the (x, μ^2) plane ($\mu^2 = Q^2$) is divided into four regions as shown in fig.7 and different parametrisation of parton distributions is assumed separately in each region. For large μ^2 ($\mu^2 > \mu_0^2$) the regions of large and small x (regions I and III in fig.7) are divided by the boundary curve $\mu_B^2(x)$

$$\mu_B^2(x) = 2 + 0.053^2 \exp \left(3.56 \sqrt{\ln \frac{1}{3x}} \right). \quad (66)$$

In the region I of large μ^2 and large x the structure functions F_2 is given by the QCD improved parton model with the scale dependent quark distributions $f_{v,s}(x, \mu^2)$ where the subscripts v and s denote the valence and sea distributions. The distributions $f_{v,s}(x, \mu^2)$ satisfy the Altarelli-Parisi evolution equations. In the regions II–IV the structure function F_2 is again expressed in terms of the quark distributions $\hat{f}_{v,s}(x, \mu^2)$ with their μ^2 dependence being different in different regions separately. In the region II of small μ^2 and large x ($x > x_0$) the structure functions are damped by the μ^2 dependent factors and valence distributions are made harder, i.e.

$$\hat{f}_v(x, \mu^2) = \left(\frac{\mu^2 \mu_0^2 + m_R^2}{\mu_0^2 \mu^2 + m_R^2} \right)^{(1-\eta)(1-x)/(1-x_0)} f_v(x, \mu_0^2) \quad (67)$$

$$\hat{f}_s(x, \mu^2) = \left(\frac{\mu^2 \mu_0^2 + m_P^2}{\mu_0^2 \mu^2 + m_P^2} \right)^{1+\epsilon} f_s(x, \mu_0^2) \quad (68)$$

In the region III of large μ^2 ($\mu_0^2 < \mu^2 < \mu_B^2(x)$) and small x , the sea and valence quark distributions are parametrised as below:

$$x \hat{f}_v(x, \mu^2) = N_1 \left(\frac{x}{x_0} \right)^\eta x_0 f_v(x_0, \mu_0^2) + N_2 x f_v(x, \mu_B^2(x)) \quad (69)$$

$$x \hat{f}_s(x, \mu^2) = N_1 \left(\frac{x}{x_0} \right)^{-\epsilon} x_0 f_s(x_0, \mu_0^2) + N_2 x f_s(x, \mu_B^2(x)) \quad (70)$$

where

$$N_1 = \frac{\ln(\mu_B^2/\mu^2)}{\ln(\mu_B^2/\mu_0^2)} \quad (71)$$

$$N_2 = 1 - N_1 \quad (72)$$

Finally in the region IV of small x and small μ^2 the valence and sea quark distributions are parametrised in the Regge form with the appropriate damping factors which guarantee vanishing of the structure function F_2 in the limit $\mu^2 = 0$

$$x \hat{f}_v(x, \mu^2) = \left(\frac{x}{x_0} \right)^\eta \left[N_1 \left(\frac{\mu_0^2 + \mu_R^2}{\mu_0^2} \right)^{1-\eta} x_0 f_v(x_0, \mu_0^2) + N_2 x_0^\eta N^v \right] \quad (73)$$

$$x \hat{f}_s(x, \mu^2) = \left(\frac{x}{x_0} \right)^{-\epsilon} \left[N_1 \left(\frac{\mu_0^2 + \mu_R^2}{\mu_0^2} \right)^{1+\epsilon} x_0 f_s(x_0, \mu_0^2) + N_2 x_0^{-\epsilon} N^s \right] \quad (74)$$

where now

$$N_1 = 1 - \frac{x_0 - x}{x_0} \frac{\mu_0^2 - \mu^2}{\mu_0^2} \quad (75)$$

$$N_2 = 1 - N_1 \quad (76)$$

The parameters ϵ and η correspond to the (effective) intercepts of the Pomeron and the Reggeon respectively ($\epsilon = 0.56$, $\eta = 0.45$). The values of the remaining parameters appearing in the formulae (67) - (75) are as follows: $\mu_0^2 = 5 \text{ GeV}^2$, $x_0 = 0.0069$, $m_R^2 = 0.92 \text{ GeV}^2$, $m_P^2 = 0.38 \text{ GeV}^2$, $N_d^v = 0.121$, $N_u^v = 2N_d^v$, $N_d^s = N_u^s = N_d^s = N_u^s = 0.044$ and $N_s^s = N_{\bar{s}}^s = 0.5N_d^s$. The structure function $F_2(x, Q^2)$ is expressed in terms of the quark (and antiquark) effective distribution functions $\hat{f}_i(x, Q^2)$ using the parton model formula:

$$F_2(x, Q^2) = x \sum_i e_i^2 \hat{f}_i(x, Q^2). \quad (77)$$

Comparison of different structure function parametrisations is shown in fig.8.

7 Nuclear shadowing

Nuclear effects in the deep inelastic lepton scattering may be experimentally inferred from inspecting the ratio F_2^A/F_2^d . In this method one tacitly assumes that the nuclear effects in F_2^d can be neglected or that $F_2^d = F_2^N$ where F_2^N describes a free nucleon. In particular the nuclear shadowing corresponds to the F_2^A/F_2^d ratio being smaller than unity at small

x . In this sense shadowing can be regarded as a part of the so called EMC effect as illustrated in fig.9.

The nuclear shadowing is a firmly established experimental fact, cf.sec.8, which has been observed both in the low Q^2 (including the photoproduction, i.e. $Q^2 = 0$) and in the large Q^2 region. The nuclear structure function F_2^A is then:

$$F_2^A = F_2^N - \delta F_2^A \quad (78)$$

where δF_2^A denotes the shadowing term ($\delta F_2^A > 0$).

In the region of low Q^2 (and for photoproduction) the natural and presumably the dominant mechanism of nuclear shadowing is the multiple scattering of vector mesons in the nucleus, fig.10. The vector mesons couple to virtual (or real) photons. This model gives the following contribution of the shadowing to the nuclear structure function:

$$\delta F_2^A = \frac{Q^2}{4\pi} \sum_v \frac{M_v^4 \delta \sigma_v^A}{\gamma_v^2 (Q^2 + M_v^2)^2} \quad (79)$$

where the cross section $\delta \sigma_v^A$ is that part of the vector meson – nucleus total cross section σ_v^A (normalised to a nucleon) which corresponds to multiple scattering i.e.:

$$\sigma_v^A = \sigma_v^N - \delta \sigma_v^A. \quad (80)$$

Here σ_v^N is the vector meson – nucleon total cross section and the remaining quantities in the formula (79) are defined in sec.3. The cross section which corresponds to the multiple scattering can be obtained from the Glauber theory [48, 10]. The negative sign in eq.(80) reflects the fact that the vector meson–nucleon scattering amplitude is assumed to be imaginary. Thus the shadowing in the inelastic lepton–nucleus scattering, in the VMD reflects the absorptive character of the elementary vector meson–nucleon scattering amplitude. It follows from eq.(79) that the shadowing term which corresponds to the multiple rescattering of (finite number of) vector mesons vanishes for large Q^2 .

At large Q^2 one expects the parton model to be applicable. It leads to the Bjorken scaling mildly violated by the perturbative QCD corrections. The parton model is described by the "hand–bag" diagram of fig.11. It is this "hand–bag" structure and the point-like coupling of the photons to partons (i.e. to quarks and antiquarks) which guarantees the Bjorken scaling (modulo the perturbative QCD corrections) independently of the structure of the lower part of the diagram. The nuclear shadowing in the large Q^2 region may come from the multiple interaction contributions to the lower part of the hand–bag diagram as shown in fig.11. Different models of shadowing correspond to different structure details of the diagrams of fig.11 (see [49]).

Shadowing is expected to be a low x phenomenon. This can be understood within the simple space–time picture of the interaction of the virtual photon with atomic nucleus, [50]. In the infinite momentum frame i.e. in a frame where the momentum $p = p_A/A$ is very large the wee partons (sea quarks and gluons) occupy longitudinal distances

$$\Delta z_p \approx \frac{1}{xp} \quad (81)$$

The momentum p_A is the momentum of the nucleus with A nucleons. The nucleus in this frame occupies the Lorentz contracted distance:

$$\Delta z_A \approx 2R_A \frac{M}{p} \quad (82)$$

where R_A is the nuclear radius and M the nucleon mass. The (Lorentz contracted) average distance between the nucleons in this frame is:

$$\Delta z = r \frac{M}{p} \quad (83)$$

where r denotes the average distance between nucleons in the laboratory frame. One can distinguish three regions in x , i.e.:

$$(i) \quad x > \frac{1}{Mr} \quad (84)$$

that corresponds to the partonic size being smaller than the average distance between nucleons within nuclei:

$$\Delta z_p < \Delta z. \quad (85)$$

In this region the shadowing is expected to be negligible since partons can be regarded as belonging to individual nucleons.

$$(ii) \quad \frac{1}{2MR_A} < x < \frac{1}{Mr} \quad (86)$$

that corresponds to the longitudinal size of the partons being larger than average distance between nucleons in a nucleus yet smaller than the (contracted) longitudinal size of the nucleus:

$$\Delta z_A > \Delta z_p > \Delta z. \quad (87)$$

In this region the shadowing gradually sets on with the decreasing x .

$$(iii) \quad x < \frac{1}{2MR_A} \quad (88)$$

that corresponds to

$$\Delta z_p > \Delta z_A \quad (89)$$

where shadowing is expected to be maximal. In the regions (i) and (ii) the partons can no longer be regarded as belonging to individual nucleons since their longitudinal distances are larger than the average distances between nucleons exceeding eventually the size of the nucleus.

Let us also notice that when the deep inelastic scattering is considered in the laboratory frame then the three regions of x , i.e. (i) – (iii) are defined through the mutual relations between the lifetime τ_γ of the $q\bar{q}$ fluctuation of the virtual photon:

$$\tau_\gamma = \frac{1}{Mx} \quad (90)$$

and the characteristic distances r and R_A . The nuclear shadowing indeed is a typical small x phenomenon since $1/(Mr) \approx 0.1$.

There exist several models of shadowing incorporating the VMD and/or the partonic mechanisms. For a complete review of these models and a comprehensive list of references see ref. [49]. Since that article has been completed new papers concerning the shadowing in the deuteron have appeared [51, 52, 53], several of them inspired by the new measurements of the F_2^d/F_2^p ratio (cf. sec.8). Understanding of shadowing effects in the deuteron has become relevant in view of the increased precision of the measurements of the F_2^A/F_2^d and F_2^d/F_2^p ratios. In particular the shadowing affects extraction of the neutron structure function from the data. This in turn affects (decreases) the magnitude of the experimentally evaluated Gottfried sum ([54, 51]).

8 Experimental data

The experimental problems connected to measuring and analysing the low x , low Q^2 data are discussed in [6]. In that reference we have also listed the experiments which provided the then available experimental results. Here we limit ourselves only to updating this information.

In the fixed target experiments the low x region is correlated with the low Q^2 values, see e.g. fig.12, [55]. The lowest values of x were reached by the NMC at CERN and E665 Collaboration at Fermilab through applying special experimental techniques permitting measurements of muon scattering angles as low as 1 mrad, [56, 57, 55, 58]. These "small x triggers" and special off-line selection methods were also effective against a background of muons scattered elastically from target atomic electrons; corresponding peak occurs at $x = 0.000545$. Systematic errors in both experiments, in particular these on the ratio of structure functions for different nuclei, F_2^a/F_2^b , were greatly reduced as a result of irradiating several target materials at a time and/or of a frequent exchange of targets in the beam.

During the last three years there appeared new measurements of the $F_2^p(x, Q^2)$, $F_2^d(x, Q^2)$ [1] and $F_2^d(x, Q^2)/F_2^p(x, Q^2)$ [29, 59, 56, 57] by the NMC and by the E665 [60, 58]. The NMC also performed the QCD analysis of their F_2 data [4]. Both collaborations have presented new and precise results on x , A and Q^2 dependence of nuclear shadowing [56, 55, 61, 62]. The NMC also determined differences $R^{Ca} - R^C$, [63] and $R^d - R^p$, [64] at low x .

Extraction of $F_2(x, Q^2)$ from the data needs information on $R(x, Q^2)$. In particular the ratio of inelastic cross sections on different nuclei is equal to the corresponding structure functions ratio, provided $R(x, Q^2)$ is the same for these nuclei. Results of the NMC analysis of $R^{Ca} - R^C$ and $R^d - R^p$ show that neither of these quantities exhibit a significant dependence on x and that they are both compatible with zero, fig.13. In their analyses the NMC and E665 assumed R independent of the target atomic mass A .

The NMC results for the deuteron structure function, F_2^p as a function of Q^2 for different bins of x , are shown in fig.14. Measurements cover ranges $0.006 \leq x \leq 0.6$ and $0.5 \leq Q^2 \leq 55 \text{ GeV}^2$ and were the first precise measurements at such low values of x . The data had great impact on the parton distribution analysis (see e.g. [44]) and join well to the results of HERA, [66]. A clear scaling violation pattern with slopes $d \ln F_2 / d \ln Q^2$ positive at low x and negative at higher x and an "approach to scaling" (i.e. Q^2 dependence of F_2 at Q^2 less than few GeV^2) are visible in fig.14. In this figure comparison of the NMC [1], SLAC [34] and BCDMS [35] measurements is also shown. The agreement between all three data sets is very good. At the same time the low x results of the EMC NA2 [67] experiment have been disproved by the NMC measurements. The x dependence of the deuteron structure function at low Q^2 is shown in fig.15 for several values of Q^2 . The NMC measurements are compared with those of EMC NA28 [36] and SLAC [34]. Characteristic is a weak x dependence of F_2^d at low x .

The Q^2 dependence of the structure functions F_2^p and F_2^d measured by the NMC with good accuracy down to low values of x has been compared with the predictions of perturbative QCD [4]. The flavour singlet and non-singlet quark distributions as well as the gluon distribution have been parametrised at the reference scale equal to 7 GeV^2 . All the data with $Q^2 \geq 1 \text{ GeV}^2$ were included in the fit. Besides the leading twist contribution the higher twist term was also included in an approximate way given by formula (15) where $H(x)$ was determined from the SLAC and the BCDMS measurements [68], averaged over

the proton and deuteron and suitably extrapolated to lower values of x . Results of the QCD fit to the proton structure function data are shown in fig. 16. Important here is the extension of the QCD analysis to the low x and low Q^2 regions. The contribution of higher twists is still moderate at scales about 1 GeV².

The new (preliminary) measurements of the proton and deuteron structure functions for $x > 0.0001$ have recently been presented by the E665 Collaboration, fig.17 [58]. The lowest Q^2 values in their data reaches a very small value of 0.2 GeV².

Both NMC and E665 experiments have measured the deuteron to proton structure function ratio, F_2^d/F_2^p , extending down to very low values of x . In case of the NMC the ratio has been measured directly, i.e. the measurement of the absolute structure function is used only for the radiative corrections calculations. The data are usually presented as the ratio F_2^n/F_2^p where F_2^n is defined as $2F_2^d - F_2^p$. This quantity would give the structure function of the free nucleon in the absence of nuclear effects in the deuteron. NMC results for F_2^n/F_2^p as the function of x are shown in fig. 18, [57]; E665 results are presented in fig. 19, [58]. In the latter case instead of the ratio F_2^n/F_2^p the ratio σ_n/σ_p is given. It is equal to the former due to A -independence of R . In both data sets the average Q^2 varies from bin to bin reaching down to $\langle Q^2 \rangle = 0.2$ GeV² at $x = 0.0008$ for the NMC and $\langle Q^2 \rangle = 0.004$ GeV² at $x = 5 \times 10^{-6}$ for E665. The results of both experiments show that the F_2^n/F_2^p stays always below unity down to the smallest measured values of x which at low x can be attributed to the nuclear shadowing in the deuteron [51, 52, 53].

New data appeared on the nuclear shadowing. Preliminary results of a high precision study of the A dependence of nuclear shadowing by the NMC performed in the range $0.01 < x < 0.7$ and $2 \text{ GeV}^2 < Q^2 < 60 \text{ GeV}^2$, are shown in figures 20–22. The structure function ratios F_2^A/F_2^C for $A = \text{Be, Al, Fe and Sn}$ together with earlier data of SLAC [69], show a detailed pattern of the x dependence of shadowing, fig.20. The NMC data cover the A range from $A = 2$ to $A = 118$. In fig.21 they are shown as a function of A for three bins of x . The functional dependence of F_2^A/F_2^C on A is approximately logarithmic (except, possibly, the light nuclei) and the structure function ratio has therefore been parametrised as $F_2^A/F_2^C = a + b \ln A$ in each bin of x . The slopes, b , are displayed in fig.22. The amount of shadowing increases strongly with the mass number A . Much lower values of x and Q^2 are covered by the nuclear data obtained by the E665: $x > 0.0001$ and $Q^2 > 0.1 \text{ GeV}^2$ thanks to a special trigger and off-line analyses, [62], fig.23. Shadowing seems to saturate at x about 0.004 as was observed earlier by the E665 [61] and indicated by the preliminary NMC data on the F_2^{Li}/F_2^D and F_2^C/F_2^D ratios measured down to $x = 0.0001$ and $Q^2 = 0.03 \text{ GeV}^2$, [55]. No clear Q^2 dependence is visible in the E665 data in a wide interval of Q^2 , fig.24.

9 Conclusions and outlook

In this review paper we have summarised the present understanding of the electroproduction structure functions in the region of low values of x and Q^2 , which have recently been measured in the fixed target, charged lepton inelastic scattering experiments. This has included a survey of theoretical constraints and expectations, clarification of certain definitions and concepts, collection of the existing parametrisations of structure functions in this region and presentation of the experimental data. We consider this paper as being an extension of the comprehensive review article on the small x physics [6] towards the more detailed treatment of the low Q^2 problems. This means in particular that in the

summary of the experimental data we have presented only the results which appeared after the above mentioned article had been completed.

Important property of the structure function $F_2(x, Q^2)$ which follows from the conservation of the electromagnetic current is its linear vanishing as the function of Q^2 for $Q^2 \rightarrow 0$ (for fixed ν). This means that Bjorken scaling cannot be a valid concept at low Q^2 and so the pure partonic description of inelastic lepton scattering has to break down for moderate and low Q^2 values. At moderately large Q^2 the higher twist contributions to F_2 which vanish as negative powers of Q^2 are frequently being included in the QCD data analysis. One also expects that at low Q^2 the VMD mechanism should play an important role.

The small x behaviour of the structure function $F_2(x, Q^2)$ is dominated by the pomeron exchange. Analysis of the structure function in the small x region for both low and moderate values of Q^2 can clarify our understanding of the pomeron, i.e. possible interplay between the "soft" and "hard" pomerons, the role of shadowing (or absorptive) corrections, etc. At large Q^2 the problem is linked with the QCD expectations concerning the deep inelastic scattering at small x , see ref.[70] for a recent review. Besides the structure functions (or total cross sections) complementary information on the pomeron can also be obtained from the analysis of diffractive processes in the electro- and photoproduction. This concerns both the inclusive diffractive processes and the diffractive production of vector mesons; the present experimental situation is described in refs. [66, 71, 72, 73].

Descriptions of the low Q^2 , low x behaviour of F_2 range from pure fits to experimental data to the dynamically motivated models. The existing parametrisations have been collected in a way which should make it easy to use in practical applications, e.g. in the radiative correction procedure. Wherever possible a dynamical content of each parametrisation was exposed.

Since 1992 a wealth of measurements of F_2 has been published. In the region of interest for this paper this included the NMC and E665 results extending down to very low x and Q^2 values and displaying characteristic "approach to scaling" behaviour. Nuclear shadowing phenomenon was studied in great detail for targets ranging from $A = 2$ to $A = 118$ by the same two collaborations. Its x , Q^2 and A dependence was precisely measured.

The new possibilities have opened up with the advent of the HERA collider. Presently the data there are collected at Q^2 larger than approximately 5 GeV^2 and therefore have not been discussed in this review. These data show a very strong increase of F_2 with decreasing x , [37, 38]. Let us remind that for fixed Q^2 the increase of the structure function $F_2(x, Q^2)$ with decreasing x reflects the increase of the virtual Compton scattering total cross section with W^2 . We would like to emphasise that this increase observed at large Q^2 is much stronger than the increase of the total photoproduction cross section observed between the fixed target and the HERA energies. Dynamical understanding of this evident variation of the W^2 dependence with Q^2 is certainly very interesting and important. The experimental data at low values of Q^2 ($Q^2 \simeq 1 \text{ GeV}^2$) which would cover similar range of W^2 as those at high Q^2 would be extremely valuable for this purpose. It would therefore be very interesting to extend the HERA measurement down to the low values of Q^2 as it has been planned recently [74].

10 Acknowledgments

We thank our colleagues from the NMC for numerous discussions and from the E665 collaboration for providing us with their data. This research has been supported in part by the Polish State Committee for Scientific Research grant number 2 P302 062 04.

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Figure Captions

1. Illustration of the continuity of physical processes: double differential cross section for the electron–proton inelastic scattering is sketched as a function of the energy transfer ν for different values of the resolution Q^2 . Dashed and continuous lines correspond to constant values of x and W respectively. Definitions of kinematical variables are given in sec.2.
2. Kinematics of inelastic charged lepton–proton scattering in the one photon exchange approximation and its relation through the optical theorem to Compton scattering for the virtual photon.

3. Total photoproduction cross section as a function of the γp centre of mass energy, $W_{\gamma p}$. Low energy data come from ref.[20], recent ZEUS Collaboration data – from ref.[22], previous data – from ref.[21]. The curves are explained in the text. Figure comes from ref.[22].
4. Parametrisation of $F_2^p(x, Q^2)$ by Donnachie and Landshoff as a function of Q^2 for the following values of x (from the top): 0.008, 0.035, 0.07, 0.18 and 0.45 together with data from NMC [1], SLAC [34] and BCDMS [35]. The F_2 values are scaled by factors (from the top): 16, 8, 4, 1.5 and 1 for clarity. The error bars represent statistical and systematic errors added in quadrature (from [2]).
5. Parametrisation of $F_2^p(x, Q^2)$ by Abramowicz et al. The x values, data and scaling factors are as in fig.4 (from [2]).
6. Parametrisation of $F_2^p(x, Q^2)$ by Badelek and Kwieciński as a function of Q^2 for the following values of x (from the top): 0.008, 0.0125, 0.0175, 0.035, 0.07 and 0.1 together with data from NMC [1], SLAC [34] and BCDMS [35]. The F_2 values are scaled by factors (from the top): 32, 26, 8, 4, 2 and 1 for clarity. The error bars represent statistical and systematic errors added in quadrature (from [2]).
7. Regions of the (x, μ^2) plane as used in the parametrisation by Schuler and Sjöstrand (from [18]).
8. Comparison of the parametrisations of $F_2^p(x, Q^2)$ by Abramowicz et al. (full curves), Badelek and Kwieciński (dashed curves) and Donnachie and Landshoff (dash-dotted curves) as functions of Q^2 for the following values of x (from the top): 0.008, 0.02, 0.05 and 0.1. The values are scaled by the factors (from the top): 27, 9, 3 and 1 for clarity (from [2]).
9. Sketch of the x dependence of the ratio F_2^A/F_2^d where various nuclear effects are indicated.
10. Multiple scattering of vector mesons. Lines in the lower part of the diagram denote nucleons.
11. Hand-bag diagram for the virtual Compton scattering on a nucleus and its decomposition into a multiple scattering series. Lines in the upper parts of diagrams denote quarks (antiquarks) and lines in the lower part denote nucleons.
12. Kinematic ranges covered by different triggers in the NMC (from [55]).
13. NMC (preliminary) results $R^d - R^p$ as a function of x compared with the QCD predictions (the curve, see [64] for details) and with the results of SLAC (open symbols, [65]). Figure comes from ref. [64].
14. The F_2^d data from NMC compared to the data from SLAC [34] and BCDMS [35]. The error bars are the quadratic sums of the statistical and systematic errors, excluding the overall normalisation uncertainty. The curve is a result of a 15-parameter function fit to all three data sets, given in Table 1 (from [1]).
15. Comparison of the (low Q^2) x dependence of the NMC [1], EMC NA28 [36] and SLAC [34] results (from [1]).

16. Results of the QCD fit to the F_2^p data. The solid line is the result of the QCD fit with higher twist included, cf. eq. (15). The dotted curve shows the contribution of F_2^{LT} . In the fit 90 (280) GeV data were renormalised by 0.993 (1.011). The errors are statistical (from [4]).
17. Preliminary measurements of $F_2^p(x, Q^2)$ by the E665 Collaboration at Fermilab. The errors are statistical. Systematic errors are 5–15 % (from [58]).
18. NMC results on the ratio F_2^n/F_2^p as the function of x at Q^2 values averaged in each x bin. The solid symbols mark the final data set while the open ones are still preliminary data taken with a special "small x trigger". Errors are statistical; the band at the bottom indicates the preliminary estimate of systematic errors (from [57]).
19. E665 results for σ_n/σ_p as a function of x at Q^2 values averaged in each x bin for three methods of data analysis. Errors are statistical; the total systematic uncertainty is less than 3.5% in all x regions. In this figure the NMC data at $Q^2 = 4 \text{ GeV}^2$ are also shown (from [58]).
20. NMC results on structure function ratios, F_2^A/F_2^C , as functions of x together with the earlier results of SLAC [69] (from [56]). Errors are statistical.
21. A dependence of the NMC F_2^A/F_2^C data for three x bins (from [56]).
22. Slopes b from the $F_2^A/F_2^C = a + b \ln A$ fit to the NMC data (from [56]).
23. E665 results on σ^A/σ^D as a function of x . Errors are statistical, the systematic ones are marked as shaded bands. The 3.4% overall normalisation error has not been included (from [62]).
24. Q^2 dependence of the E665 nuclear data in bins of x (from [62]).

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